Turbulence & Wind Energy

Emmanuel Plaut, Mathieu Jenny & Michael Hölling

Session - Man	Date	Content
1 - EP	07/12	RANS approach : models 1: generalities, $k-\varepsilon$ model, begin. of pb. 1.1
2 - EP	14/12	RANS approach : models 2: middle of pb 1.1
→ 3 - MJ	04/01	RANS approach and models 3: end of pb 1.1
4 - MH	07/02	Wind resources - Conversion principles - Aero
5 - MH	08/02	Aero - Stochastic (Langevin) power curve
6 - MH	09/02	Wind field and Turbulence
MJ, EP & MH	10/02	**Exam:** Turbulence & Wind Energy
MH	10/02	**General conf.:** Turbulence & Wind Energy Research

- RANS = Reynolds Averaged Navier-Stokes
- During sessions 1 - 3, with EP, use of **Matlab** on your laptop.
- During sessions 4 - 6, with MH, use of **R** on your laptop.

http://emmanuelplaut.perso.univ-lorraine.fr/twe

RANS approach with the eddy-viscosity assumption

$$\underbrace{v_i}_{\text{velocity component}} = \underbrace{\overline{v_i}}_{\text{average } U_i} + \underbrace{u_i}_{\text{fluctuations}}$$

$$\underbrace{p}_{\text{modified pressure}} = \underbrace{\overline{p}}_{\text{average } P} + \underbrace{p'}_{\text{fluctuations}}$$

into the **Navier-Stokes equation** for a newtonian incompressible fluid

$$\rho\left[\partial_t v_i + \partial_{x_j}(v_i v_j)\right] = \partial_{x_j}(\sigma_{ij}) = -\partial_{x_i} p + \partial_{x_j}(\tau_{ij})$$

with the Cauchy stress tensor $\sigma_{ij} = -p\delta_{ij} + \tau_{ij}$

where the viscous-stress tensor $\tau_{ij} = 2\eta S_{ij}(\mathbf{v})$,

η = dynamic viscosity, $S_{ij}(\mathbf{v}) = \frac{1}{2}\left[\partial_{x_j}(v_i) + \partial_{x_i}(v_j)\right]$ = strain-rate tensor

$\rightarrow$ **Reynolds equation**

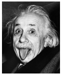
$$\rho\left[\partial_t U_i + \partial_{x_j}(U_i U_j)\right] = -\partial_{x_i} P + \partial_{x_j}(2\eta S_{ij}(\mathbf{V})) + \partial_{x_j}(\tau_{ij})$$

with the **Reynolds-stress tensor** $\tau_{ij} = -\rho\,\overline{u_i u_j} = -\rho$ covariance$(v_i v_j)$

Boussinesq EVA: $\exists$ **eddy viscosity** η_t s.t.

$$\tau_{ij} = -\frac{2}{3}\rho k\delta_{ij} + 2\eta_t S_{ij}(\mathbf{V})$$

with the **turbulent kinetic energy** (per unit mass) $k = \dfrac{1}{2}\overline{u_i u_i}$



RANS Launder & Spalding $k - \varepsilon$ model

→ **Reynolds-Boussinesq equation** or **RANS momentum equation**

$$\partial_t U_i \;+\; \partial_{x_j}(U_i U_j) \;=\; -\frac{1}{\rho}\partial_{x_i}\Pi \;+\; 2\partial_{x_j}[(\nu + \nu_t)S_{ij}(\mathbf{V})]$$

with corrected pressure $\Pi \;=\; P \;+\; \frac{2}{3}\rho k$ that implies the **turbulent kinetic energy**

$$\boxed{k \;=\; \frac{1}{2}\overline{u_i u_i}}\;.$$

It fulfills

$$\partial_t k \;+\; U_j \partial_{x_j} k \;=\; \partial_{x_j}(\nu \partial_{x_j} k) \;-\; \partial_{x_j}\left(\frac{1}{\rho}\overline{p' u_j} \;+\; \frac{1}{2}\overline{u_i u_i u_j}\right) + (\partial_{x_j} U_i)\frac{\tau_{ij}}{\rho} - \varepsilon$$

with the **turbulent dissipation**

$$\boxed{\varepsilon \;=\; \nu\overline{(\partial_{x_j} u_i)(\partial_{x_j} u_i)}}\;.$$

Idea: the **eddy viscosity** is controlled by **these two quantities**,
by dimensional analysis

$$\nu_t \;=\; \eta_t/\rho \;=\; C_\nu \; v_k \; \ell_{k\varepsilon} \;=\; C_\nu \; k^2/\varepsilon\;.$$

RANS Kolmogorov then Wilcox $k - \omega$ model

→ **RANS momentum equation**

$$\partial_t U_i + \partial_{x_j}(U_i U_j) = -\frac{1}{\rho}\partial_{x_i}\Pi + 2\partial_{x_j}[(\nu + \nu_t)S_{ij}(\mathbf{V})]$$

with corrected pressure $\Pi = P + \frac{2}{3}\rho k$ that implies the **turbulent kinetic energy**

$$\boxed{k = \frac{1}{2}\overline{u_i u_i}} \ .$$

It fulfills

$$\partial_t k + U_j \partial_{x_j} k = \partial_{x_j}(\nu \partial_{x_j} k) - \partial_{x_j}\left(\frac{1}{\rho}\overline{p' u_j} + \frac{1}{2}\overline{u_i u_i u_j}\right) + (\partial_{x_j} U_i)\frac{\tau_{ij}}{\rho} - \varepsilon$$

with the **turbulent dissipation**

$$\boxed{\varepsilon = \nu\overline{(\partial_{x_j} u_i)(\partial_{x_j} u_i)}} \ .$$

Idea: relevant quantity = **rate of dissipation of turbulence per unit energy** or **turbulence dissipation frequency**

$$\boxed{\omega = \varepsilon / k} \implies \text{by dimensional analysis } \nu_t = C_\nu \ k/\omega \ .$$

Back to the Launder & Spalding $k - \varepsilon$ model

- Equation for **V** : **RANS momentum equation**:

$$\partial_t U_i \; + \; \partial_{x_j}(U_i U_j) \; = \; -\frac{1}{\rho}\partial_{x_i}\Pi \; + \; 2\partial_{x_j}[(\nu + \nu_t)S_{ij}(\mathbf{V})] \; .$$

- Exact equation for k :

$$\partial_t k \; + \; U_j \partial_{x_j} k \; = \; \partial_{x_j}(\nu \partial_{x_j} k) \; - \; \underbrace{\partial_{x_j}\left(\frac{1}{\rho}\overline{p' u_j} \; + \; \frac{1}{2}\overline{u_i u_i u_j}\right)}_{\text{unknown} \;\Rightarrow\; \text{modeled with } \nu_t} \; + \; P_k \; - \; \varepsilon$$

with the **production term** deduced from the Boussinesq hyp.

$$P_k \; = \; (\partial_{x_j} U_i)\frac{\tau_{ij}}{\rho} \; = \; 2\nu_t S_{ij}(\mathbf{V})S_{ij}(\mathbf{V}) \; > \; 0$$

Modeled equation:

$$\partial_t k \; + \; U_j \partial_{x_j} k \; = \; \partial_{x_j}[(\nu + \nu_t)\partial_{x_j} k] \; + \; P_k \; - \; \varepsilon \; .$$

Launder & Spalding $k - \varepsilon$ model

- Equation for $\mathbf{V}$: **RANS momentum equation**:

$$\partial_t U_i \;+\; \partial_{x_j}(U_i U_j) \;=\; -\frac{1}{\rho}\partial_{x_i}\Pi \;+\; 2\partial_{x_j}[(\nu+\nu_t)S_{ij}(\mathbf{V})]\;.$$

- Equation for k :

$$\partial_t k \;+\; U_j\partial_{x_j}k \;=\; \partial_{x_j}[(\nu+\nu_t)\partial_{x_j}k] \;+\; P_k \;-\; \varepsilon$$

with the **production term** $P_k \;=\; 2\nu_t S_{ij}(\mathbf{V})S_{ij}(\mathbf{V})$.

- Equation for ε obtained by analogy with the k - eq. + dimensional analysis:

$$\partial_t\varepsilon \;+\; U_j\partial_{x_j}\varepsilon \;=\; \partial_{x_j}[(\nu+\sigma_\varepsilon^{-1}\nu_t)\partial_{x_j}\varepsilon] \;+\; \frac{\varepsilon}{k}(C_1 P_k \;-\; C_2\varepsilon)\;.$$

Launder & Spalding $k - \varepsilon$ model

$$\frac{DU_i}{Dt} = \partial_t U_i + U_j \partial_{x_j} U_i = -\frac{1}{\rho}\partial_{x_i}\Pi + 2\partial_{x_j}[(\nu+\nu_t)S_{ij}(\mathbf{V})]$$

$$\frac{Dk}{Dt} = \partial_t k + U_j \partial_{x_j} k = \partial_{x_j}[(\nu+\nu_t)\partial_{x_j} k] + P_k - \varepsilon$$

$$\frac{D\varepsilon}{Dt} = \partial_t \varepsilon + U_j \partial_{x_j} \varepsilon = \partial_{x_j}[(\nu+\sigma_\varepsilon^{-1}\nu_t)\partial_{x_j} \varepsilon] + \frac{\varepsilon}{k}(C_1 P_k - C_2 \varepsilon)$$

$$P_k = 2\nu_t S_{ij}(\mathbf{V})S_{ij}(\mathbf{V}) \; , \quad \nu_t = C_\nu \; k^2/\varepsilon$$

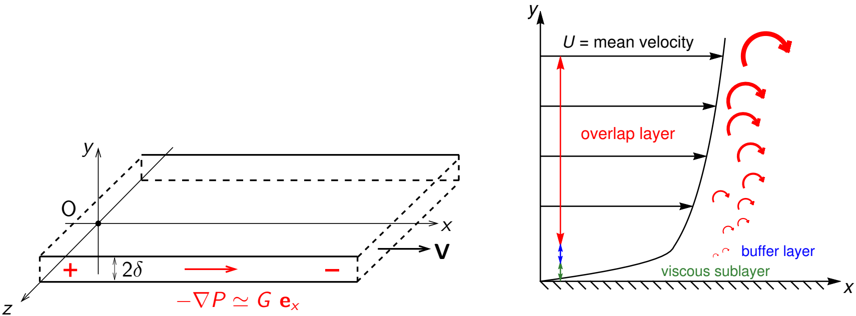
Coefficient	σ_ε	C_1	C_2	C_ν	Von Karman constant χ
'Universal' (?) standard value	1.3	1.44	1.92	0.09	0.433

- In a **plane jet** or **mixing layer**, the rate of growth of the flow $\rightarrow$ C_1
- In **turbulence behind a grid**, the rate of decay of the flow $\rightarrow$ C_2
- In the **overlap layer**, **wall laws** with a χ value $\rightarrow$ C_ν and σ_ε

[Launder & Spalding 1974 *Comp. Meth. Appl. Mech. Eng.*]

Pb 1.1: learn about turbulent flows, the mixing-length and $k-\varepsilon$ RANS models, by focussing on channel flows

General setup: Focus near the lower wall:



Mean velocity $\mathbf{V} = U(y)\ \mathbf{e}_x$, mean (modified) pressure $P = -Gx + p_0(y)$,

all other mean fields depend only on y.

Pb 1.1: results of part 1: generalities and mixing-length model

- Integrated RANS momentum eq. in the x-direction :

$$\eta \frac{dU}{dy} + \tau_{xy} = \tau_w - Gy$$

with the **mean wall shear stress**

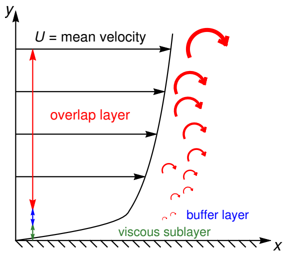
$$\tau_w = \eta \frac{dU}{dy}\Big|_{y=0} = G\ \delta\ .$$

- **Viscous sublayer**: y small, viscous stress dominates

$$\eta \frac{dU}{dy} \simeq \tau_w \quad \Longrightarrow \quad U \simeq U_0' y$$

$$\Longleftrightarrow \quad U^+ = \frac{U}{u_\tau} = y^+ = \frac{y u_\tau}{\nu}$$

with u_τ the **friction velocity** such that $\tau_w = \rho u_\tau^2$.



Pb 1.1: results of part 1: generalities and mixing-length model

- Integrated RANS momentum eq. in the x-direction :

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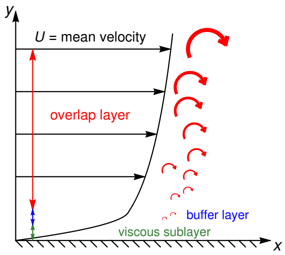


- **Log layer**: y larger, Reynolds stress dominates, which is estimated with a **mixing-length model**

$$\tau_{xy} \simeq \eta_t \frac{dU}{dy} \simeq \tau_w\ , \quad \eta_t \simeq \rho \frac{dU}{dy} (\chi y)^2$$

$$\iff \quad U^+ = \frac{U}{u_\tau} = \frac{1}{\chi} \ln y^+ + C$$

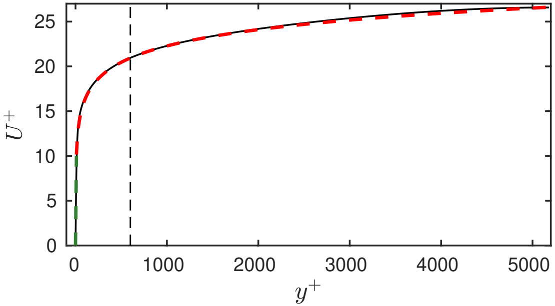
with u_τ the **friction velocity** such that $\tau_w = \rho u_\tau^2$.

NB: $\eta_t = \rho u_\tau \chi y \iff \nu_t = u_\tau \chi y \iff \nu^+ := \frac{\eta_t}{\eta} = \chi y^+$.

Pb 1.1: results of parts 2 and 3: comparisons with the UT DNS

DNS data at $Re_\tau = 5200$ display a **viscous sublayer** for $y^+ \lesssim y_0^+ = 4$ and a **log layer** around $y^+ \simeq 600$...

- **viscous sublayer profile** $U^+ = y^+$ relevant up to $y^+ = 10$
- **log layer profile** $U^+ = \chi^{-1} \ln y^+ + C$, with $\chi = 0.38$ and $C = 4.1$, relevant from $y^+ = 10$ to $y^+ = \delta^+$



- $\Rightarrow$ **Karman - Prandtl theory** for the friction factor...
- **Universality of this three-layers structure**...

Part 4: Analytical verification of the $k - \varepsilon$ model in the log layer

- There Reynolds stress dominates in the integrated RANS momentum U_x eq.

$$\eta \frac{dU}{dy} + \tau_{xy} = \tau_w - Gy \quad \Longrightarrow \quad \tau_{xy} = \eta_t \frac{dU}{dy} \simeq \tau_w = \rho u_\tau^2 \; .$$

- **Mixing length model**

$$\eta_t \simeq \rho \frac{dU}{dy} (\chi y)^2 \quad \Longrightarrow \quad U^+ = \frac{U}{u_\tau} = \frac{1}{\chi} \ln y^+ + C$$

$$\Longrightarrow \quad \eta_t = \rho u_\tau \chi y \quad \Longleftrightarrow \quad \nu_t = u_\tau \chi y \quad \Longleftrightarrow \quad \nu^+ := \frac{\eta_t}{\eta} = \chi y^+ \; .$$

4.1 Assume $k - \varepsilon$ model is valid + production - dissipation balance in the k eq.

$$\Longrightarrow \quad \textbf{wall laws} \quad \text{for} \quad k \text{ and } \varepsilon \; : \quad k = \frac{u_\tau^2}{\sqrt{C_\nu}} \; , \quad \varepsilon = \frac{u_\tau^3}{\chi \; y} \; .$$

4.2 $\Longrightarrow$ **Townsend's relation** : $-\dfrac{\overline{u_x u_y}}{k} = \text{constant} = \sqrt{C_\nu}$.

4.3 ε eq. $\Longrightarrow$ relation between σ_ε , C_1 , C_2 and C_ν : $\chi^2 = (C_2 - C_1)\, \sigma_\varepsilon \sqrt{C_\nu}$.

Part 5: Study of $\overline{u_i u_j}$, k and ν_t on the DNS database

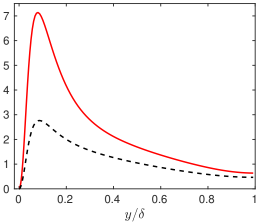
5.1 Eddy-viscosity assumption

$$\overline{u_i u_j} = \frac{2}{3} k \delta_{ij} - 2\nu_t S_{ij}(\mathbf{V})$$

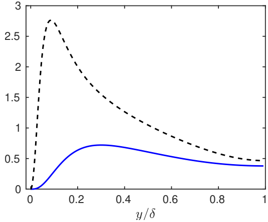
$\Longrightarrow$ **velocity variances**

$$\overline{u_x^+ u_x^+} = \frac{\overline{u_x u_x}}{u_\tau^2} = \overline{u_y^+ u_y^+} = \overline{u_z^+ u_z^+} = \frac{2}{3} k^+ \quad \text{with} \quad k^+ = \frac{k}{u_\tau^2}$$

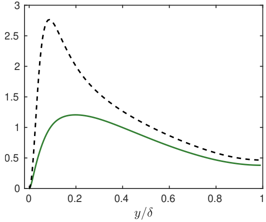
For the DNS at $Re_\tau = 180$, plots of $\overline{u_x^+ u_x^+}$, $\overline{u_y^+ u_y^+}$, $\overline{u_z^+ u_z^+}$ and $\frac{2}{3} k^+$:



wall-enhancing effect



strong wall-damping effect



wall-damping effect

Part 5: Study of $\overline{u_i u_j}$, k and ν_t on the DNS database

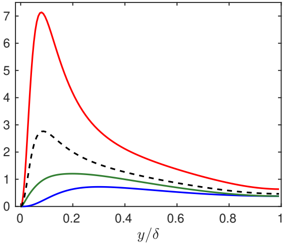
5.1 Eddy-viscosity assumption

$$\overline{u_i u_j} = \frac{2}{3} k \delta_{ij} - 2\nu_t S_{ij}(\mathbf{V})$$

$\Longrightarrow$ **velocity variances**

$$\overline{u_x^+ u_x^+} = \frac{\overline{u_x u_x}}{u_\tau^2} = \overline{u_y^+ u_y^+} = \overline{u_z^+ u_z^+} = \frac{2}{3} k^+ \quad \text{with} \quad k^+ = \frac{k}{u_\tau^2}$$

For the DNS at $Re_\tau = 180$, plots of $\overline{u_x^+ u_x^+}$, $\overline{u_y^+ u_y^+}$, $\overline{u_z^+ u_z^+}$ and $\frac{2}{3} k^+$:



Near the wall: **anisotropy** of the **velocity variances** (second moments)
that could be captured only by **'second moments'** or **'Reynolds stress models'** !

Part 5: Study of $\overline{u_i u_j}$, k and ν_t on the DNS database

5.2 Eddy-viscosity assumption $\Longrightarrow$ **velocity covariances**

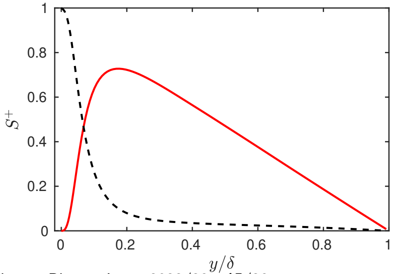
$$\overline{u_x^+ u_z^+} = \overline{u_y^+ u_z^+} = 0 \,, \quad \overline{u_x^+ u_y^+} = -\nu^+ S^+$$

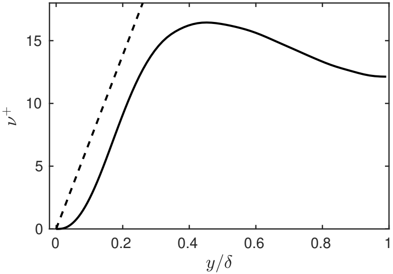
with the **mean strain rate** $S^+ = \dfrac{dU^+}{dy^+}$.

5.3 For the DNS $Re_\tau = 180$

$$\max\left(\max\left|\overline{u_x^+ u_z^+}\right| = 0.0058 \,, \quad \max\left|\overline{u_y^+ u_z^+}\right| = 0.0013\right) \ll \left(\max\left|\overline{u_x^+ u_y^+}\right| = 0.728\right) .$$

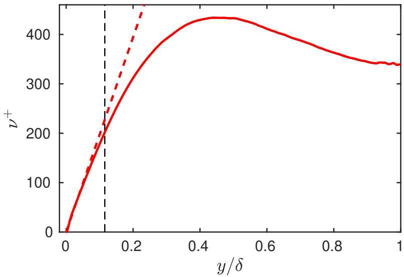
5.4 For the DNS $Re_\tau = 180$, plots of S^+, $-\overline{u_x^+ u_y^+}$
and the **'exact' eddy viscosity** $\nu^+ = -\overline{u_x^+ u_y^+}/S^+$ with the **log-layer-law** $\nu^+ = \chi y^+$:

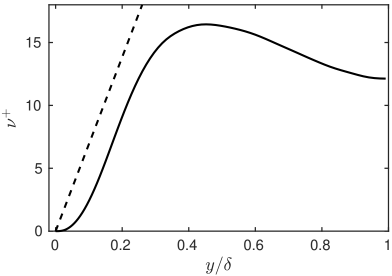




Part 5: Study of $\overline{u_i u_j}$, k and ν_t on the DNS database

5.5 Compare the DNS $Re_\tau = 5200$ to the DNS $Re_\tau = 180$:

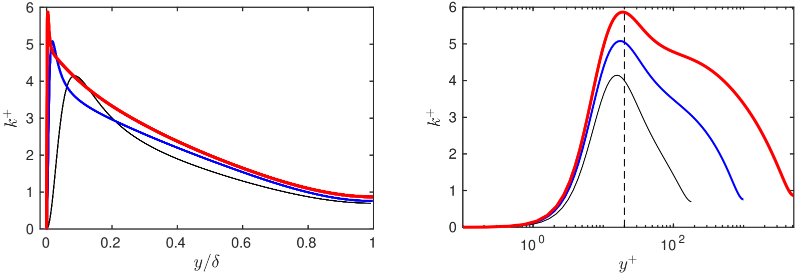




- **eddy viscosity** much **higher** because **turbulence** is **stronger**
- **log-layer-law** $\nu^+ = \chi y^+$ now **relevant** !

Part 6: Study of k, P_k and ε on the DNS database

6.1 For the DNS $Re_\tau = 180$, 1000 and 5200,
the **turbulent kinetic energy** k^+ vs y/δ in lin-lin scales, vs y^+ in log-lin scales:



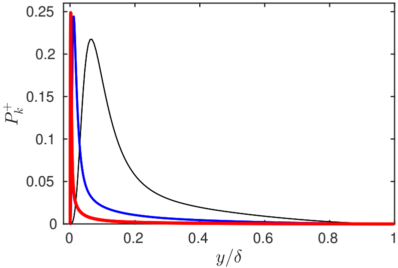
- at fixed y^+, $k^+ \uparrow$ as $Re_\tau \uparrow$
- k^+ peaks at a robust value $y^+_{\max k} \simeq 20$
- max k^+ does not converge as $Re_\tau \to \infty$?

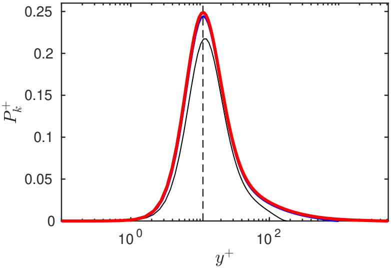
6.2 The **log-layer law** or **wall law** for k

$$k^+ \;=\; \frac{1}{\sqrt{C_\nu}} \quad \text{is} \quad \textbf{irrelevant !}$$

Part 6: Study of k, P_k and ε on the DNS database

6.3 For the DNS $Re_\tau = 180$, 1000 and 5200,
the **production term** $P_k^+ = -\overline{u_x^+ u_y^+}\ S^+$ vs y/δ in lin-lin scales, vs y^+ in log-lin scales:





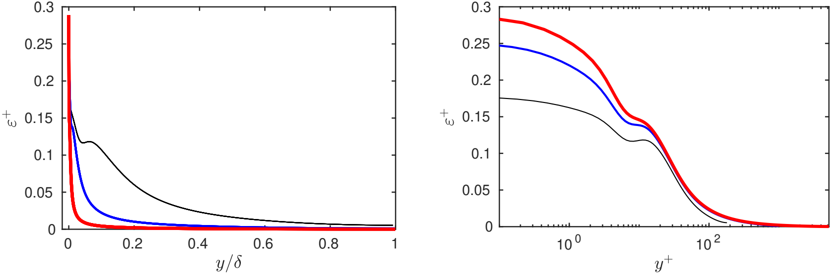
- at fixed y^+, $P_k^+ \uparrow$ as $Re_\tau \uparrow$
- P_k^+ peaks at a robust value $y^+_{\max P_k} \simeq 11$
- max P_k^+ converges as $Re_\tau \to \infty$
- $\exists$ **asymptotic profile** for $Re_\tau \to \infty$?

A **general asymptotic theory** for the main covariance $\overline{u_x u_y}$, the mean strain rate S and, consequently, P_k and ν_t , as $Re_\tau \to \infty$, has been recently presented... see the pb.1.3 !..

[Heinz 2018, 2019 On mean flow universality of turbulent wall flows. Parts I & II. *J. Turbulence*]

Part 6: Study of k, P_k and ε on the DNS database

6.4 For the DNS $Re_\tau = 180$, 1000 and 5200,
the **turbulent dissipation rate** ε^+ vs y/δ in lin-lin scales, vs y^+ in log-lin scales:



ε^+ larger in the near-wall region !

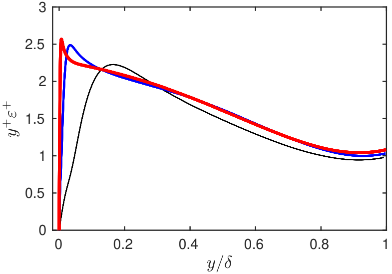
6.5 The value of ε^+ at the wall is **not** universal **!**

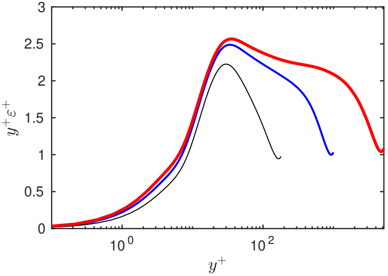
Part 6: Study of k, P_k and ε on the DNS database

6.6 For the DNS $Re_\tau = 180$, 1000 and 5200,
the **premultiplied turbulent dissipation rate** $y^+\varepsilon^+$,
which should be according to the **log-layer law** or **wall law**

$$y^+\varepsilon^+ \;=\; \chi^{-1} \;\simeq\; 2.5 \quad \text{with} \quad \chi \;\simeq\; 0.4\;,$$

vs y/δ or y^+ :



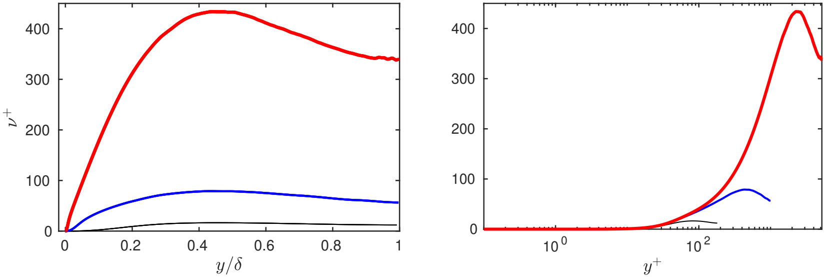


$\Longrightarrow$ the **log-layer law** or **wall law** for ε is **poorly relevant !**

Part 7: Study of ν_t , k^2/ε and C_ν on the DNS database

7.1 Dimensionless form of the $k-\varepsilon$ **model eddy-viscosity** $\boxed{\nu^+ \;=\; C_\nu \;\dfrac{k^{+2}}{\varepsilon^+}}$.

7.2 For the DNS $Re_\tau = 180$, 1000 and 5200,
the **'exact' eddy viscosity** $\nu^+ = -\overline{u_x^+ u_y^+}/S^+$ vs y/δ or y^+ :



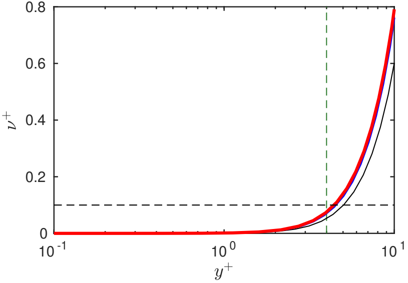
Part 7: Study of ν_t , k^2/ε and C_ν on the DNS database

7.1 Dimensionless form of the $k - \varepsilon$ **model eddy-viscosity** $\boxed{\nu^+ \;=\; C_\nu \; \frac{k^{+2}}{\varepsilon^+}}$.

7.3 Zoom on the range $y^+ \in [0.,10]$ to check that, in the **viscous sublayer**, defined by

$$y^+ \;\lesssim\; y_0^+ \;=\; 4 \;,$$

the **'exact' eddy viscosity** $\nu^+ < 0.1$:

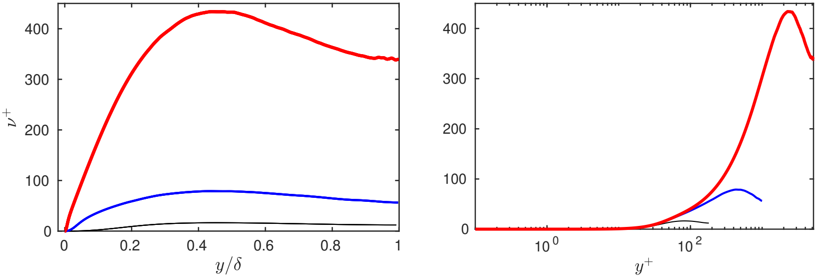


$\Longrightarrow$ there the **eddy viscosity** plays no role !

Part 7: Study of ν_t , k^2/ε and C_ν on the DNS database

7.1 Dimensionless form of the $k-\varepsilon$ **model eddy-viscosity** $\boxed{\nu^+ = C_\nu \frac{k^{+2}}{\varepsilon^+}}$.

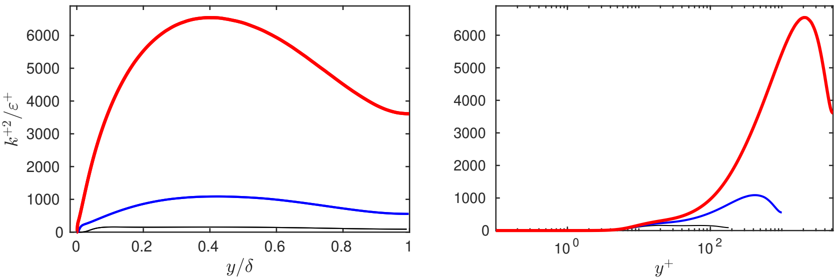
7.2 For the DNS $Re_\tau = 180$, 1000 and 5200,
the **'exact' eddy viscosity** ν^+ vs y/δ or y^+ :



Part 7: Study of ν_t , k^2/ε and C_ν on the DNS database

7.1 Dimensionless form of the $k-\varepsilon$ **model eddy-viscosity** $\boxed{\nu^+ \;=\; C_\nu \; \frac{k^{+2}}{\varepsilon^+}}$.

7.4 For the DNS $Re_\tau = 180$, 1000 and 5200,
the ratio k^{+2}/ε^+ vs y/δ or y^+ :



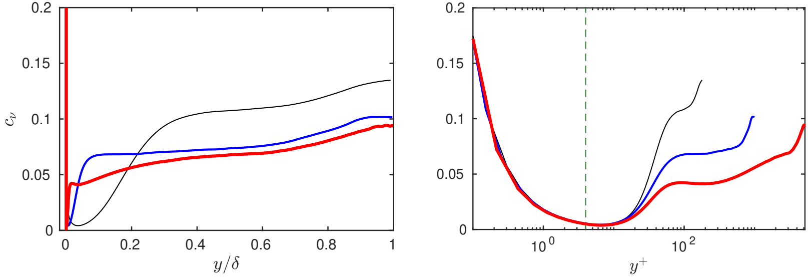
Part 7: Study of ν_t , k^2/ε and C_ν on the DNS database

7.1 Dimensionless form of the $k-\varepsilon$ **model eddy-viscosity** $\boxed{\nu^+ = C_\nu \frac{k^{+2}}{\varepsilon^+}}$.

7.5 For the DNS $Re_\tau = 180$, 1000 and 5200,

the **ideal eddy viscosity 'function'** c_ν such that $\nu^+_{\text{exact}} = c_\nu \frac{k^{+2}}{\varepsilon^+}$

vs y/δ or y^+, with the limit $y_0^+ = 4$:

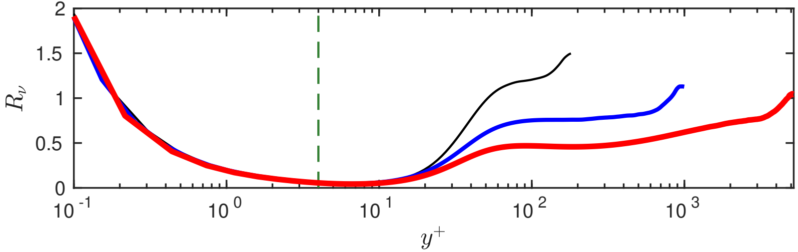


Part 7: Study of ν_t , k^2/ε and C_ν on the DNS database

7.1 Dimensionless form of the $k-\varepsilon$ **model eddy-viscosity** $\boxed{\nu^+ = C_\nu \frac{k^{+2}}{\varepsilon^+}}$.

7.5 For the DNS $Re_\tau = 180$, 1000 and 5200,

the **eddy viscosity ratio** $R_\nu := \dfrac{\nu^+_{\text{exact}}}{0.09\ k^{+2}/\varepsilon^+}$ vs y/δ or y^+, with the limit $y_0^+ = 4$:

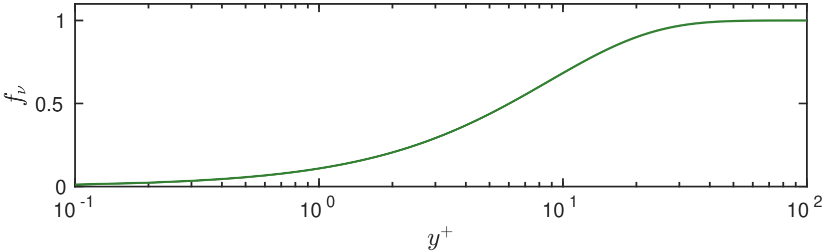


Irrelevant viscous sublayer	**wall-damping effect**	kind of 'plateau'

Comments on the pb. 1.1 regarding the $k - \varepsilon$ model

- ☹ **'Wall laws'** for k and ε in the **log-layer** are **poorly relevant**:
  bad news for the **'high-Reynolds number' model**
  where one would start computing in the **log-layer**, around $y^+ \simeq 100$...
- ☹ The **eddy-viscosity law** is **poorly relevant**, especially, in the near-wall region !
- ☺ **'Low-Reynolds number' models** exist, where one starts computing at the wall $y = 0$, and uses **'damping functions'** e.g. in the **eddy-viscosity**

$$\nu_t \;=\; C_\nu \, f_\nu \, \frac{k^2}{\varepsilon} \quad \text{with} \quad f_\nu \;=\; f_\nu(y^+) \;=\; 1 - \exp(-0.115 y^+)$$



[Chien 1982 revisited by Hanjalić & Launder 2011]

Openings: many RANS models exist !..

The **Turbulence Modeling Resource** web site of NASA Langley Research Center

https://turbmodels.larc.nasa.gov

lists 18 models, among which 3 **One-Equation Models** which are somehow ν_t **models**:

- the **Spalart-Allmaras model**,
- the **Nut-92 model**,
- the **Wray-Agarwal model**...

There has been also recently proposals for $k - \nu_t$ **models**, e.g. by Menter and coworkers; these models may run in **hybrid RANS-LES mode** which looks promising !..

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http://emmanuelplaut.perso.univ-lorraine.fr/twe