Module *Advanced Fluid Mechanics* by E. Plaut

Homework 2 - New final version

Linear stability properties of Viscous Plane Poiseuille Flow at high Reynolds number

The solution of this Homework should take the form of a Notebook NAME.nb that you will upload on the ARCHE page of the module, in the section devoted to this purpose, before Friday February 3 at 17:00 in the afternoon (compulsory deadline). This Notebook should be clearly presented with some comments; the graphs should be clearly labelled.
As stated in the title of the Homework, we want to study the stability of Viscous Plane Poiseuille Flow at high Reynolds number; this exercise is therefore a follow-up of Ex. 2.2.

1 With the program that you wrote for Ex. 2.2, using at least $N_z = 18$ spectral modes, plot the real part of the eigenvalue $\sigma(k, R)$ of the most relevant mode with a streamwise wavenumber

$$k = 0.9$$

versus $R \in [5\ 10^3, 100\ 10^3]$. Observe that this mode is amplified only in a range of Reynolds number $R \in [R_0(k), R_1(k)]$.

2 Code with `FindRoot` the computation of the high Reynolds number $R_1(k)$ at which $\mathrm{Re}[\sigma(k, R)]$ vanishes.

3 Saving the value of $R_1(k)$ to files with names `R1Nz*` and re-reading these files, implement the convergence criterion that $R_1(k)$ does not change by more than 0.01% with $N_z - 1$ vs N_z modes: if $r_{1\text{low}} = R_1(k; N_z - 1)$ and $r_1 = R_1(k; N_z)$, one wants that

$$|r_1/r_{1\text{low}} - 1| \ < \ 10^{-4} \ .$$

Determine according to this criterion the lowest value of N_z that one should use for this study at high Reynolds number, and an estimate of $R_1(k)$ with 2 digits.
Importantly, you should observe that you run into precision problems at high N_z. To increase the precision, define the collocation points with

```
z[m_]= N[Cos[m Pi/(2 Nz+1)],Nz]
```

Hereafter we do no more focus on $k = 0.9$ but explore a range of values of k.

4 With the value of N_z determined in question 3, construct a list `lkR0` of the couples $(k, R_0(k))$ with k the wavenumber, $R_0(k)$ the lower neutral Reynolds number, for discrete k values spanning the interval

$$0.7 \ \leq \ k \ \lesssim \ 1.1 \ .$$

Construct also a list `lkR1` of the couples $(k, R_1(k))$ with k the wavenumber, $R_1(k)$ the higher neutral Reynolds number, for discrete k values spanning the interval

$$0.8 \ \leq \ k \ \lesssim \ 1.1 \ .$$

To compute this second list, use a continuation method: start at large k and decrease k by small steps, using as an estimate of $R_1(k)$ in the `FindRoot` command the value of $R_1(k)$ computed at the previous step.
Join these two lists and plot in linear - log scales the limit of the region where TS waves are amplified in the (k, R) plane, in the intervals

$$0.7 \ \leq \ k \ \leq \ 1.1 \ , \quad 5\ 10^3 \ \leq \ R \ \leq \ 200\ 10^3 \ .$$

5 Explain what happens as $R \to +\infty$ using a theoretical result valid in this limit.